

Two-Sided Platforms with Endogenous Quality Differentiation

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(Preliminary. Comments welcome)

Abstract

In this paper we construct a simple model of platform price competition with two main novel features. First, platforms endogenously decide the quality of their ‘access service’ and second, each group exhibits preferences not only about the number of agents in the opposite group, but also about their type or quality. Additionally, sellers also care about the type of agents in their own group. Our interest is to examine the set of conditions under which, in spite of the network externalities, more than one platform coexist in equilibrium. We show that when quality is endogenously determined by the choices of agents these platforms could be asymmetric.

Keywords: two-sided markets, platforms, networks, vertical differentiation.

JEL Classification: L10, L13, D40

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1. Introduction

Nowadays two-sided platforms are present in many aspects of our life. When we pay with the debit or the credit card in the gas station, when we search for a flight ticket or a hotel in the web, when we buy a newspaper or simply when we go to the shopping mall, we are having access or using a platform that allows us to connect in a particular way with agents on the other side of the market. In general, the more numerous these agents are the larger is our interest on the platform.¹ However, in many cases the choice of the platform is also conditioned by other circumstances like the identity of the agents that we are going to meet, the brands that are offered inside the platform or our level of income. Think of malls in a big city and sellers and buyers visiting them. On the sellers' side, we observe that some brands are present in all of them whereas others are not. On the buyers' side, buyers choose the mall they visit according to the sellers they are thinking to buy from. At the same time, expensive brands have preferences about the type of buyers and locate in malls visited by high income people. Finally, sellers belonging to the group of expensive brands choose to group together in the same malls, although it makes competition between them stronger.² Consequently, we may think that heterogeneity of the customers on each side of the market would play an important role in the formation of the platforms, the type of platform that is going to arise and the prices that they can set.

The recent and growing literature on two-sided platforms have largely considered models with network externalities in which members of each customer group benefit if more members of the other customer group are on the same platform.³ However, in many markets, agents' decisions also affect the level of quality offered by the platform and then others customers' utilities, so that, a quality externality takes place. The existence of this externality may help us to explain why, although the presence of strong network externalities, it is not common to observe concentration in industries based on two-sided platforms. As Evans et al. (2005) remarks, product differentiation may be an important countervailing force working against concentration.

This paper proposes a model that allows for the two types of externalities, the standard indirect network effect and the externality because of the quality concern. There are two sides of the market, buyers and sellers. Sellers are of two types, the sellers that offer a product of high quality and the sellers that offer a low quality product. For the low type sellers we maintain the traditional assumption that they only care about the mass of buyers that participate in their platform. In contrast, we assume that high type sellers care about the

¹Note what the advertisement of Mastercard says: "There are some things money can not buy, for everything else there is Mastercard".

²To buy clothing in Madrid, for instance, a possible choice is between going to the Village outlet, where we meet Loewe or Versace, or going to the Factory outlet, to buy from Zara or Mango. A similar phenomena is present in online travel platforms. In some of them only cheap airlines participate (see for instance, Terminal A, Cheap Flights), whereas big or expensive airlines participate in a different group of platforms (Lastminute, Travelprice).

³See the classical papers of Armstrong (2004), Rochet and Tirole (2003, 2004).

mass of buyers of a particular type and about the type of members on its own side.^{4,5} Buyers are heterogeneous and care about the mass and type of sellers.

The first goal is to determine the conditions under which the network effect is stronger than the quality effect so that concentration prevails (i.e., there is a single active platform in equilibrium) and the conditions to have equilibria with more than one active platform. We find that a sufficient condition to have a concentrated market is that high type sellers do not care about the other sellers in the platform.

A second goal is, working with a model where ex-ante platforms are equal, to determine the conditions by which market equilibria can be characterized by two platforms having different qualities, number of agents and prices.⁶ Platforms' quality is endogenously determined by the type of sellers they house, so that the level of quality is increasing in the proportion of sellers of high quality.⁷ We find that depending on parameter values there are equilibria where sellers separate by type, equilibria where every seller multihome, and equilibria where low type singlehome whereas high type multihome.

We find that in any equilibrium where sellers singlehome and separate by type, at least one type of sellers make zero profits. In addition, profits of the platform that houses the low quality sellers are higher or equal than profits of the platform housing high quality sellers. For this equilibrium to exist, the high quality sellers must strongly care about partners in the platform, and the mass of lowest types of buyers must be large enough. We also find that there are equilibria where low type sellers singlehome while high type multihome. We show that in any of them profits of both platforms must be equal, although their qualities are different. Finally, another kind of equilibrium that may emerge is one with two identical platforms housing all the sellers each of them. Interestingly, the prices that they set and their profits could be different.

Two close papers to this one are Gabszewicz and Wauthy (2004) and Damiano and Li (2005). In the first one, authors model competition between two platforms within a vertical differentiation framework. However, only network effects matter in agents' utility functions given that they assume that quality in one market is endogenously determined by the size of the network in the other market. In other words, the platform with the larger number of members on side i , is seen by members on side j as a good of higher quality than the other platform. In our model qualities are determined by the sellers' products itself.

The model of Damiano and Li (2005) considers participants that care about the identities of other participants and quality of the matching is endogenously

⁴Pashigian and Gould (1998) analyse empirically the demand externalities existing among stores in a shopping mall. They argue that there exist "anchor stores" (well known stores) that create external economies to other stores.

⁵Any own's side effect has been largely ignored in the literature. An exception is Nocke, Peitz and Stahl (2004) that assume competition between sellers.

⁶A common feature in the literature is the presence of symmetric equilibria with identical competing platforms setting the same prices, see Rochet and Tirole, 2003, and Armstrong and Wright, 2004, Gabszewicz, et. al. (2001).

⁷Papers leading with platforms that differ in the normal Hotelling way include Armstrong (2004), Armstrong and Wright (2004), Gabszewicz, et. al. (2004).

determined. Although this important similarity, there are three main differences with our model. First, agents do not care about the mass in the other side, this is, there is no network externalities between customers. Second, agents of the two sides are assumed that do not care about members on its own side. Finally, in their model customers on each side singlehome while we allow the sellers to multihome. These differences have important consequences on price competition and the equilibria configurations that may arise. Whenever platforms compete in prices simultaneously, their model only presents equilibria in mixed strategies, whereas we find conditions under which pure strategies equilibria indeed exist. Moreover, because sellers can multihome, competition between platforms is less fierce in our model. In addition, in the Damiano and Li's model an equilibrium with a single active platform never arises, while we find that because of the network effects, this is the unique equilibrium when sellers do not care about partners in the platform.

The paper is organized as follows. In the next section, we present the model. In section 3, we analyze equilibria considering that charges are zero to the buyers. In section 4, we search for the equilibria when charges are zero to the sellers. In section 5, we study some welfare considerations. Finally, we conclude in section 6.

2. The model

We study platform price competition in an environment with endogenous vertical differentiation. There are two ex-ante identical platforms operating in a two-sided market. One side is a measure one of sellers and the other side is a measure one of buyers. Platforms offer an access service that provides to each side the possibility of connecting with agents on the other side. This service conveys two characteristics for each side of the market: the quality of the platform and the number (mass) of agents on the other market's side participating in the platform. The platforms set a charge to permit the access and then, endogenously determine the characteristics of the service offered.

Buyers and sellers' decisions consist in paying for the access to the platforms or not. In particular, buyers are allowed to access to only one platform (singlehome) while sellers can access to both of them simultaneously (multihome). We are thinking in platforms such that at a point of time a seller or a brand can be present in more than one platform while a buyer has to choose one of them to visit (malls are a good example).

Buyers are heterogeneous in the value they assign to the platform's quality and homogeneous in the corresponding value to the network. We model these facts with a net utility function (utility net of price) of the form

$$U_i^{B\theta} = \theta q^i + \gamma N_i^S - P_i^B \quad (1)$$

where q^i is the quality of platform i and θ measures how much buyer θ values this quality. The network parameter is γ and the total number of sellers in platform

i is represented by N_i^S . The price that platform i charges to the buyers is P_i^B . Note that quality of the platform and mass of sellers are substitutes in the function⁸. Beyond the heterogeneity, all the buyers are more attracted by both, the platform that houses the larger number of sellers and the platform with the highest quality. Heterogeneity determines differences in the weights to each utility component⁹.

There are two type of sellers, the high type H, with measure x , and the low type L with measure $1 - x$ (assume that $x < \frac{1}{2}$). The quality of a platform depends on the number of high type sellers relative to the total of sellers in the platform, so that its value belongs to the interval $q^i \in [0, 1]$. In particular, it takes value $q^L = 0$ when the platform houses only low type sellers and value $q^H = 1$ when the platform accounts only for high type sellers. If the platform houses all sellers its quality is $q^M = x$.

We assume that θ is distributed according to a Burr type XII distribution¹⁰ with parameter λ :

$$\theta \sim F(\theta) = 1 - \left[1 - \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \right]^{\lambda} \quad ; \lambda \geq 0, \quad \theta \in \Theta = [\underline{\theta}, \bar{\theta}] \quad \text{and} \quad \underline{\theta} \geq 0.$$

The value of λ identifies the level of concentration around high or low values of θ . In particular, if $\lambda = 1$, θ is uniformly distributed. If $\lambda > 1$, high valuation consumers are more numerous than low valuation consumers and the opposite occurs if $\lambda < 1$. If $\lambda = 0$, distribution becomes degenerate at $\theta = \underline{\theta}$. For simplicity, we assume $\bar{\theta} - \underline{\theta} = 1$.

We define the mass of sellers (sellers' demand) on each platform according to

$$N_i^S = N_i^H + N_i^L$$

where N_i^H and N_i^L are the mass of sellers of type H and type L in platform i , respectively. We denote the mass of buyers (buyers' demand) that visit the platform i by N_i^B .

The net utility of of sellers that only go to platform i is equal to

$$U_i^s = V^s(q^i, N_i^B) + N_i^B (\beta^s) - P_i^S \quad s = H, L \quad (2)$$

and the utility of sellers that multihome is equal to $U_i^s + U_j^s$.

Note that sellers are heterogeneous in two dimensions. First, in the value they assign to platform's quality modeled by the function $V^s(q_i, N_i^B)$. The sellers of the high type care about their partners in the platform, which can be

⁸Notice that our representation of buyers' population and of their preferences is reminiscent of the well-known model of vertical product differentiation. At the same time, is a more general case of the Armstrong's model with membership fees.

⁹It may be better understood if it is interpreted as a heterogeneity in income instead of preferences. A priori every buyer value the high quality products but before visiting the platform they anticipate the purchases they can do, so that θ is the result of a problem previously solved by the buyer (as an indirect utility function).

¹⁰Burr type XII distribution has been used by Basaluzzo et al. (2005).

interpreted as a "reputation effect". In particular, $V^H(q^i, N_i^B)$ is assumed increasing and concave in q^i and constant in N_i^B , with $V^H(q^i, N_i^B) \geq q^i$ and $V^H(0, N_i^B) = V^H(q^i, 0) = 0$. The low type sellers do not perceive any reputation effect and $V^L(q^i, N_i^B) = 0$ for all q^i, N_i^B . In what follows when we write $V(q^i)$, we are referring to $V^H(q^i, N_i^B)$ given $N_i^B > 0$, i.e., to the valuation for the platform's quality of the high type sellers.

The parameters β^H and β^L determine the type of buyers that sellers are interested in, where $N_i^B(\beta^s)$ is a function defined by

$$N_i^B(\beta^s) = P(\theta \geq \beta^s) \text{ for all } \theta \text{ s.t. } U_i^{B\theta} \geq \max\{U_j^{B\theta}, 0\}.$$

It follows that $N_i^B(\beta^s) \leq N_i^B$.

Without loss of generality we assume that $\beta^L = \underline{\theta}$ and $\beta^H \in [\underline{\theta}, \bar{\theta}]$. It implies that low quality sellers value any type of buyer, whereas high type sellers only perceive utility from the buyers in the platform whose types are in the interval $[\beta^H, \bar{\theta}]$. In other words, $N_i^B(\underline{\theta}) = N_i^B$ is the complete mass of buyers visiting platform i and $N_i^B(\beta^H)$ is the mass of buyers visiting platform i of the type that high type sellers are interested in.

We rewrite the utility functions of each type of seller, using the simplified notation described above as

$$U_i^H = V(q^i) + N_i^B(\beta^H) - P_i^S \quad (3)$$

and

$$U_i^L = N_i^B - P_i^S. \quad (4)$$

Note that, inside each group, sellers are homogeneous and that network and reputation (quality) effects are substitutes in (3).¹¹

Platforms face no cost and they can not discriminate in prices within a side of the market. Their profits are given by

$$\Pi_i = P_i^B N_i^B + P_i^S N_i^S$$

The reasoning behind our modelling strategy is similar to that in Gabszewicz and Wauthy (2004). From the viewpoint of a seller, the willingness to pay to access a platform depends on her own type and the number of additional sales this seller expect to realize in the platform. All of them are conditioned by the number and type of buyers and sellers participating in the platform. From the viewpoint of the buyers, the willingness to pay to visit the platform depends on the buyers' type and on the number of purchases that they expect to make because of the platform. It depends on the number and types of sellers housed by it.

The timing of the game is the following: in the first stage platforms set prices, in the second stage sellers observe prices and decide their locations. Finally, buyers observe sellers' locations and prices, they infer platforms' quality

¹¹The utility function of the sellers is a more general case of the Armstrong's model with an additional component of quality.

and choose the one they visit. We search for the subgame perfect equilibria of this sequential game.

We proceed to solve the game in two different settings. When only sellers are charged with a positive price and when only buyers are.

3. Charges are zero to the buyers

Consider the malls in a big city. Buyers can visit them and get the benefit of meeting several shops together but, simultaneously, they also have the access to the same shops or brands in an important street of the city at price zero. Under this situation we assume that platforms can not set positive prices to the buyers. To better understand a platform with an outside option free of charge think in a platform without it. The credit card is a typical example, there is no substitute for the service that it offers to the buyers.

We solve the game by backward induction. First we solve the buyers' problem, then we solve the sellers' subgame and finally the platforms' problem. In the third stage, each buyer takes the decision that maximizes her utility given her type. In the second stage, we search for the Strong Nash Equilibrium location of the sellers (the reasons behind the use of this equilibrium refinement will be explained in Subsections 3.2). In the first stage, the basic equilibrium concept is used: no platform has an incentive to change prices given the other's prices.

3.1 The buyers' problem

At the third stage of the game, each buyer decides if visiting platform 1 or platform 2 according to the buyer's payoff (1). We assume that the strategy of not participating in any platform yields a zero payoff to any buyer¹². Sellers have been already located in stage 2 and qualities of the platforms are known in the last stage. Buyers may hence face one of the three following possible situations: 1) Two active platforms with different qualities, 2) Two active platforms with the same quality, and 3) A single active platform.

Two active platforms with $q^i > q^j$. For each (N_i^S, N_j^S) we define θ^* as the buyer who is indifferent between visiting platform i and platform j , i.e.,

$$\theta^* = \min\{\bar{\theta}, \gamma \frac{(N_j^S - N_i^S)}{(q^i - q^j)}\}. \quad (5)$$

Consequently, the mass of buyers in platform i is given by $N_i^B = 1 - F(\theta^*)$ and the mass in platform j is given by $N_j^B = 1 - N_i^B = F(\theta^*)$.

Two particular cases are going to be important in the analysis that follows. If all sellers singlehome but separate by type, such that one platform has quality $q^H = 1$ and the other one has quality $q^L = 0$, θ^* will be equal to $\min\{\bar{\theta}, \gamma(1 - 2x)\}$ and we will denote it by θ_S^* . If low type sellers singlehome but

¹² Given that $P_i^B = 0$ for all i , the strategy of not visiting any platform is weakly dominated as $\underline{\theta}q^j + \gamma N_j^S \geq 0$, where $q^j < q^i$.

high type sellers multihome, θ^* will be equal to $\min\{\bar{\theta}, \gamma\}$ and we will denote it by θ_{HM}^* .

We introduce an additional assumption: $\gamma < \bar{\theta}$. It means that the network parameter is lower than the highest type of buyers. This is necessary to find equilibria where buyers separate by platform. Note that if, for instance, $\bar{\theta} < \gamma(1 - 2x)$, all the buyers will visit only one platform and they will never separate because the network effect would be too strong.

Two active platforms with $q^i = q^j$. Two configurations may arise. If $N_i^S > N_j^S$, then $N_i^B = 1$ and $N_j^B = 0$, whereas if $N_i^S = N_j^S$, then $N_i^B = N_j^B = \frac{1}{2}$.

A single active platform j with q^j . Every buyer will visit this platform, so that $N_j^B = 1$.

3.2 The sellers' problem

At the second stage sellers decide where to locate: at one of the two platforms, at both platforms or at none of them, once the prices have been already set in the first stage. We assume that the strategy of not participating in any platform yields a zero payoff to both types of sellers.

Sellers type s go to platform i (singlehome in i) if and only if

$$U_i^s \geq \max\{U_j^s, U_i^s + U_j^s, 0\}, \quad s = H, L$$

and they multihome if and only if

$$U_i^s + U_j^s \geq \max\{U_i^s, U_j^s, 0\}, \quad s = H, L$$

Since the sellers' decisions may induce multiple equilibria, at this stage we will concentrate on equilibria which are robust to coalitional deviations. In particular, we here search for Strong Nash Equilibria: choices by platforms for which no subgroup of sellers can deviate by changing strategies jointly in a manner that increase payoffs to all its members, given that non-members stick to their original choice.¹³

To simplify notation, we write the locations of sellers as "LiHj" to refer to a location where the sellers type L follow the strategy i and sellers type H follow the strategy j , where $i, j \in \{0, 1, 2, M\}$, i.e., sellers either do not go to any platform, go to platform 1, to platform 2 or multihome. So, for instance, L1H1, means that both type of sellers only go to platform 1; L1HM means that sellers type L go to platform 1 and type H multihome.

Further, we here introduce some additional notation which simplifies the exposition: $M = 1 - F(\beta^H)$, $T_1 = 1 - F(\gamma(1 - 2x))$, $T_2 = 1 - F(\gamma)$ and $D_q = V(q^H) - V(q^M)$. Note that M is the value that the high type sellers attach to

¹³Due to the network effects, we only need to check for deviations by the grand coalition, i.e., by all the sellers of a given type.

the network, this is, the utility that the mass of buyers with θ 's higher than β^H yield to the high type sellers. T_1 is the mass of buyers that go to the platform with the highest quality when sellers separate by type. T_2 is the mass of buyers that go to the platform with the highest quality when low type sellers go to one platform while high type sellers multihome. Note that T_1 and T_2 are well defined as $\gamma < \bar{\theta}$. Finally, D_q is the extra benefit in terms of reputation for the high type sellers of sharing the platform only with sellers of their type instead of sharing the platform also with the low type sellers.

The sellers' decisions give rise to three kind of possible equilibrium configurations: singlehoming with separation by type, multihoming and a dominant platform equilibrium. In what follows we proceed to characterize these structures. At this stage, we call platform 2 to the platform that sets the higher price, i.e., $P_2^S \geq P_1^S$.

Separating equilibrium

There is a "separating equilibrium" whenever all the sellers singlehome and separate by type in platforms so that in equilibrium there is one platform with the highest possible quality and other one with the lowest possible quality.

Proposition 1 *If $D_q < T_1$ or $1 - T_1 < V(q^M)$ there is no separating equilibrium*

Proof. First, consider the case where $\beta^H > \gamma(1 - 2x)$. In a separating equilibrium configuration as LiHj, sellers' profits are given by $U_j^H = V(q^H) + M - P_j^S$ and $U_i^L = (1 - T_1) - P_i^S$. High type sellers will not deviate whenever $V(q^H) + M - P_j^S \geq \max\{V(q^M) + M - P_i^S, V(q^H) + V(q^M) + M - P_i^S - P_j^S\}$, which requires $P_j^S - P_i^S \leq D_q$ and $P_i^S \geq V(q^M)$ to hold. Similarly, low type sellers will not deviate whenever

$$(1 - T_1) - P_i^S \geq 1 - P_j^S,$$

which requires $P_j^S - P_i^S \geq T_1$. Participation constraints impose $P_i^S \leq 1 - T_1$ and $P_j^S \leq V(q^H) + M$.

Consider now the case $\beta^H < \gamma(1 - 2x)$. Conditions for the low type sellers do not change, whereas high type sellers' profits are now given by $U_j^H = V(q^H) + T_1 - P_j^S$. High type sellers will not deviate whenever

$$V(q^H) + T_1 - P_j^S \geq \max\{V(q^M) + M - P_i^S, V(q^H) + V(q^M) + M - P_i^S - P_j^S\},$$

which requires $P_j^S - P_i^S + F(\gamma(1 - 2x)) - F(\beta^H) \leq D_q$ and $P_i^S \geq V(q^M) + F(\gamma(1 - 2x)) - F(\beta^H)$ to hold.

Hence, it follows that $V(q^H) - V(q^M) \geq T_1$ and $1 - T_1 \geq V(q^M)$ are necessary conditions to be satisfied.¹⁴ ■

¹⁴Note that conditions are more stringent when $\beta^H < \gamma(1 - 2x)$. This is due to the fact that in this case network effects for the high type sellers are stronger.

Remarks about separation and heterogeneity

Note that a necessary condition as claimed for a separating equilibrium is that the extra benefit that high type sellers obtain in a separating equilibrium compared with a situation where both types of sellers are together, measured by D_q , is larger than the cost that a separating equilibrium has on low type sellers in terms of network, measured by T_1 , the difference in potential clientele between a separating equilibrium and a situation where all sellers are together.

Thus, if $D_q = 0$ a separating equilibrium can not exist as $0 < T_1$. Furthermore, the reputation effect has to be sufficiently higher when they do not share the platform with the low type sellers than when they do. The intuition is that when sellers type H do not care enough about their partners in the platform, only network effects matter and, as expected, concentration prevails. To have a separating equilibrium it is then necessary a countervailing force as platform endogenous quality differentiation.

Note also that buyers' heterogeneity is crucial for a separating equilibrium to exist because without this heterogeneity buyers would never have an incentive to separate.

In addition, if preferences about buyers are equal for both types of sellers then again the network effect would prevail and separation is not possible. To see this, note that under $\beta^H = \theta$, in a location LiHj profits to the sellers are given by $U_j^H = V(q^H) + T_1 - P_j^S$ and $U_i^L = (1 - T_1) - P_i^S$. High type sellers will not deviate whenever

$$V(q^H) + T_1 - P_j^S \geq V(q^H) + V(q^M) + 1 - P_i^S - P_j^S$$

which requires $P_i^S \geq V(q^M) + (1 - T_1)$ to hold, condition that violates the one imposed by the participation constraint, $P_i^S \leq 1 - T_1$.

The next proposition presents the general conditions that guarantee the existence of a separating equilibrium.

Proposition 2 *Let $\beta^H > \gamma(1 - 2x)$. There is a separating equilibrium L1H2 if and only if $T_1 \leq P_2^S - P_1^S < D_q$, $V(q^M) \leq P_1^S \leq 1 - T_1$ and $P_2^S \leq V(q^H) + M$.*

Proof. The statement that a configuration of equilibrium L1H2 implies these conditions on prices follows directly from the proof of proposition 1. The implication in the other direction is shown in Appendix A. ■

The difference between the prices set by the high and the low quality platforms has to compensate the loss in terms of buyers for the sellers type L of being separated (T_1), and has to be lower than the benefit that sellers type H obtain in terms of reputation (D_q). The price in the low quality platform must be higher than $V(q^M)$ to avoid the multihoming strategy of the high type sellers. The rest of the conditions come from the participation constraints. Under symmetric conditions on prices there is a separating equilibrium like L2H1.

Equilibria involving multihoming

There are three strategies profiles that can generate an equilibrium involving multihoming. They are LMHM at which both types multihome (global multihoming from now on), LiHM with $i \neq M$, at which only high types multihome

(H multihoming from now on) and the LMHi with $i \neq M$, at which only low types multihome (L multihoming from now on). We first show that the last strategy profile is never an equilibrium configuration.

Lemma 1 There is no L multihoming equilibrium in pure strategies

Note that if sellers type H are in platform i , going to this platform generates to type L a benefit $1 - P_i^s$ while multihoming yields $1 - P_i^s - P_j^s$. So, given that type H sellers are located in one platform, the strategy LM will never be a best response. Also, if sellers type L decide to multihome, platform i will have a level of quality q^M , while the platform with only type L sellers will have quality q^L . No buyer will visit platform j given that it will have a lower quality and a lower number of sellers.

Nevertheless, global multihoming and H multihoming are equilibrium configurations as shown in next propositions.

Proposition 3 *There is a global multihoming equilibrium if and only if $P_1^S + P_2^S \leq 1$ and $P_2^S \leq \min\{T_2, V(q^M)\}$*

Proof. See the appendix A. ■

First condition comes from the participation constraint of the low type sellers. They will multihome if the benefit of doing so, i.e., the access to every buyer, at least compensate the cost of participating in both platforms. In addition, both prices must be lower than the extra benefit that multihoming generates to both types of sellers compared to the benefit of participating in only one platform (T_2 for the low type sellers and $V(q^M)$ for the high type sellers).

This result does not change if high sellers' preferences about buyers are like the ones of the low type sellers. In other words, the same result remains true if $\beta^H = \beta^L = \underline{\theta}$.

Proposition 4 *There is a H multihoming equilibrium L1HM if and only if $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$*

Proof. See the appendix A. ■

In an equilibrium like this, the quality of platform 1 is q^M while in platform 2 is q^H . To attract the low type buyers, the price in platform 1 must be lower than the one in platform 2, and P_2^S must be larger than the extra benefit (T_2) that would accrue to the low type sellers by multihoming. Note that, if low type sellers are in platform 1, the high type can ensure access to the buyers that they are interested in by locating in any of the two platform. They will participate in a second platform if it implies an extra positive pay off and so, prices in each platform must be lower than the corresponding values in terms of reputation i.e. $P_1^S \leq V(q^M)$ and $P_2^S \leq V(q^H)$. Finally, the condition $P_1^S \leq 1 - T_2$ comes from the low type sellers' participation constraint. Under the symmetric conditions on prices there is a H multihoming equilibrium L2HM.

This result remains true if high type sellers' preferences about buyers are like those of the low type.

Dominant platform equilibrium

There is a "dominant platform equilibrium" if both types of sellers single-home in the same platform. In this equilibrium there is only one active platform with medium quality.

Proposition 5 *There is a dominant platform equilibrium L1H1 if and only if $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + M\}$ and $P_2^S \geq V(q^H)$.*¹⁵

Proof. See the appendix A. ■

For high type sellers to stay in the active platform, the difference in prices must compensate the difference in terms of reputation between strategies H1 and H2. Other restrictions on P_1^S arise from the participation constraints. Finally, the condition $P_2^S \geq V(q^H)$ ensures that the high type sellers are deterred from multihoming. Under symmetric prices there is a L2H2 equilibrium.

We have here only shown the pure strategies equilibria.¹⁶ The mixed strategies equilibria are discussed in appendix B.

In figure 1 we present the map of equilibria at the sellers' stage, given the prices set by the platforms.¹⁷ The figure shows clearly the necessity of conditions in proposition 1 to have separation. As D_q and T_1 tend to get close to each other separation becomes unlikely. The same occurs when $1 - T_1$ and $V(q^M)$ get close. Whenever $\lambda > 1$, the distribution function stochastically dominates the uniform distribution so that T_1 is larger than in the case of $\lambda = 1$. The contrary occurs if $\lambda < 1$. Then, the more concentrated is the buyers' distribution around the higher types, the less likely that separation emerges. Note that the mass of high type sellers, x , affects T_1 directly, so that a smaller mass of high type sellers facilitates separation. On the contrary, changes in γ affect negatively T_1 and makes separation more likely. It may appear counterintuitive that a higher buyers' valuation for the network facilitates buyers separation. This is due to the fact that the low quality platform has a higher mass of buyers ($1 - x > x$ by assumption). Thus, if γ increases, the utility that this platform generates is higher for every buyer, and more buyers will decide to visit this platform. Then T_1 decreases, and consequently the cost for the low type sellers of being separated is smaller.

As long as T_2 is large, the global multihoming equilibrium is more likely. The impact of λ and γ on T_2 are analogous to those explained for T_1 . Recall that T_2 is the cost for the low type sellers of staying in only one platform, compared with the benefit of the multihoming strategy, when high type multihome.

For the high type sellers, the value they assign to each strategy is mainly conditioned by the value that they assign to $V(q^H)$ versus $V(q^M)$. Given that $V(q^M)$ represents the cost for the high type sellers of being separated and

¹⁵The condition $P_2^S - P_1^S \geq D_q$ is relaxed to $P_2^S - P_1^S \geq D_q - [F(\gamma(1 - 2x)) - F(\beta^H)]$ whenever $\beta^H < \gamma(1 - 2x)$.

¹⁶Note that there are other potential equilibrium configurations that we have not defined: the L0Hi and the LiH0, $i \in \{0, 1, 2, M\}$.

¹⁷To construct figure 1 we have to restrict to some conditions on parameters. In particular, we consider that conditions to have a separating equilibrium are satisfied, and $V(q^M) > T_1$.

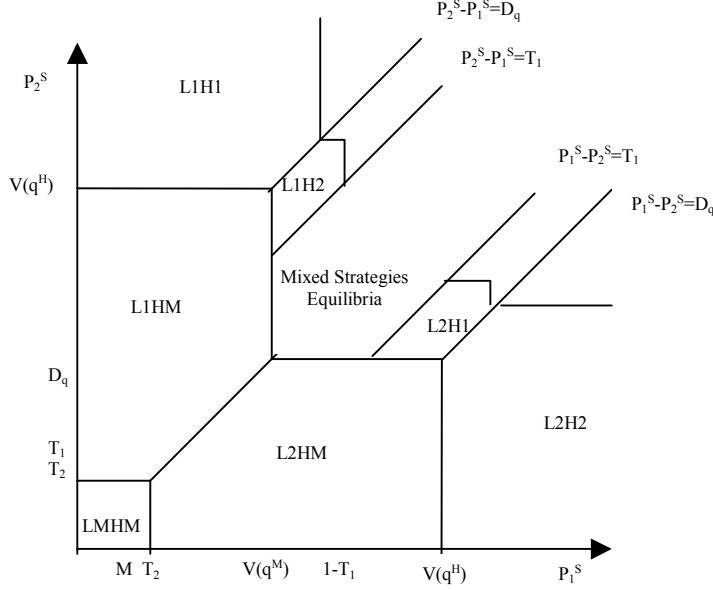


Figure 1: The Nash Equilibria at the sellers' stage

not multihoming, if $V(q^M)$ is sufficiently high, an equilibrium where low type sellers singlehome and high type multihome is more likely than a separating equilibrium. The contrary occurs if $V(q^M)$ is negligible.

3.3 Platforms' problem

Each platform i sets the price P_i^S that maximizes

$$\Pi_i = P_i^S N_i^S$$

Depending on parameters and platforms' choices, different market configurations can arise in equilibrium. In this section we are interested in the conditions under which the equilibria that we have defined and analyzed for the sellers' stage can also arise in the first stage when platforms decide prices. We concentrate in the number of platforms and the level of quality that arise in equilibrium. First, we analyze the dominant platform equilibrium where there is only a single active platform with quality q^M . Second, we present the equilibria with the two active platforms: the global multihoming where the level of quality is q^M in both platforms, the H multihoming with one platform with quality q^M and other one with quality q^H , and the separating equilibrium with qualities q^H and q^L .

A single active platform with quality q^M

The next proposition presents the necessary and sufficient conditions to have a Subgame Perfect Nash Equilibrium with a single active platform housing the total mass of sellers.

Proposition 6 *A single active platform with both types of sellers participating exists as a Subgame Perfect Dominant Platform Equilibrium if and only if $V(q^H) = V(q^M) = 0$. If $\beta^H > \gamma(1 - 2x)$ prices are $P_1^S = P_2^S = 0$. If $\beta^H < \gamma(1 - 2x)$ prices are $P_1^S = \min\{T_1, [F(\gamma(1 - 2x)) - F(\beta^H)]\}$ and $P_2^S = 0$.*

Proof. First, we prove that the existence of a single active platform as a Subgame Perfect Nash Equilibrium implies that $V(q^H) = V(q^M) = 0$.

Let $V(q^H) > V(q^M)$ and assume by contradiction that prices (P_1^S, P_2^S) are such that sellers locate at L1H1. At this candidate $\Pi_2 = 0$. Consider a deviation to $P_2^S = V(q^H) - \varepsilon$. At these prices the Nash Equilibrium is a location L1HM and $\Pi_2 > 0$, then location L1H1 can not be an equilibrium. Now, let $V(q^H) = V(q^M) = V(q) > 0$ and assume by contradiction that prices (P_1^S, P_2^S) are such that sellers locate at L1H1. At this candidate $\Pi_2 = 0$. Consider a deviation to $P_2^S = V(q) - \varepsilon$. At these prices the Nash Equilibrium is a location L1HM and $\Pi_2 > 0$, then location L1H1 can not be an equilibrium.

The statement that $V(q^H) = V(q^M) = 0$ implies a dominant platform equilibrium follows from the Bertrand's argument.

Under $\beta^H > \gamma(1 - 2x)$, if $V(q^H) = V(q^M) = 0$ both platforms yield the same benefit (gross of prices) to the sellers and there is no difference between the platforms. They compete to attract the sellers and equilibrium prices are equal to the marginal cost of the platforms. There is no profitable deviation to higher prices. Due to the network effects, even though prices are zero, sellers locate all together in one platform.

Under $\beta^H < \gamma(1 - 2x)$, if low type sellers are located in platform 1, the best reply of sellers type H is H1 as $[1 - F(\beta^H)] - P_1^S > [1 - F(\gamma(1 - 2x))] - P_2^S$.

If high type sellers are located in platform 1, the best reply of sellers type L is L1 as $1 - P_1^S > 1 - T_1 - P_2^S$.

Platform one has the market power to set a price according with these differences once it attracts one type of the sellers and it does so. In equilibrium only one platform will be active, as claimed. ■

Proposition 6 remarks the importance of the reputation effect in our model. If high type sellers do not care about their partners in the platform, there is only one possible equilibrium, the dominant platform equilibrium. The network effects are the strongest and concentration prevails.¹⁸

Two active platforms with quality $q^i = q^j = q^M$

In the following proposition we analyse the conditions to have a Subgame Perfect Equilibrium where all the sellers participate in both platforms.

¹⁸In Damiano and Li's model, where no reputation effect exists and where the network effect is absent, an equilibrium with a unique platform never arises.

Proposition 7 *If $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$ and $V(q^H)x < T_2$ there is a Subgame Perfect Global Multihoming Equilibrium with prices $P_1^{S*} = P_2^{S*} = \min\{T_2, V(q^M)\}$. If $\min\{T_2, V(q^M)\} > \frac{1}{2}$, there is a set of subgame perfect equilibria if $P_1^S + P_2^S = 1$ and $xV(q^H) < \min\{1 - P_2^S, 1 - P_1^S\}$ are satisfied.*

Proof. See the appendix C. ■

From proposition 3 it follows that P_1^{S*} and P_2^{S*} are the maximum prices that platforms can set whenever $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$ and participation constraint of the low type sellers is trivially satisfied. In this case platforms' profits are $\Pi_1^* = \Pi_2^* = \min\{T_2, V(q^M)\}$, that must be higher than the benefits that locations L1HM and L2HM would yield to the platforms, $V(q^H)x$. This fact generates the inequality $V(q^H)x < T_2$ (assumptions about $V(q^M)$ ensures that $V(q^H)x < V(q^M)$ always occurs). If the situation is that $\min\{T_2, V(q^M)\} > \frac{1}{2}$, the participation constraint will optimally be satisfied with equality. Given that profits of each platform will be $\Pi_1 = 1 - P_2^S$ and $\Pi_2 = 1 - P_1^S$, any of them should be larger than $V(q^H)x$, which explains the last inequality. Note that under condition $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$, the equilibrium, if it exists, is unique. Whereas, if $\min\{T_2, V(q^M)\} > \frac{1}{2}$, there is a set of possible equilibria. In any case $xV(q^H)$ must be necessarily lower than $\frac{1}{2}$. An interesting result here is that equilibria where both platforms provide the same service to the sellers but charge different prices and get different profits can arise, as the following corollary remarks.

Corollary *If $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$, profits of both platforms in any LMHM equilibrium are equal. If $\min\{T_2, V(q^M)\} > \frac{1}{2}$, profits will be equal if and only if $P_1^S = P_2^S = \frac{1}{2}$*

Two active platforms with $q^i > q^j$

We study now those equilibria which ensure two platforms with different qualities to arise. In particular, those where low type sellers singlehome whereas high type multihome and those where sellers separate by type. The former gives rise to a platform with medium quality and another with high quality, whereas the latter gives rise to a platform of high quality with another of low quality.

Before proceeding we present a lemma that characterizes the platforms market behavior when sellers play mixed strategies, whose proof is the content of Appendix B.

Lemma 2 *Taking the other platform price as given, the market of any platform when sellers play mixed strategies is non-decreasing (non-increasing) in its own price whenever $\frac{D_q}{T_1} \geq (\leq) \frac{1-x}{x}$.*

The market is non decreasing, i.e., $\frac{D_q}{T_1} \geq \frac{1-x}{x}$, only if $V(q^M) > T_1$, as $V(q^i)$ is a concave function.

Note that, given the price of the other platform, the market of each platform is increasing (constant) on its own price if the extra benefit of separation for high type sellers times their total mass, $D_q x$, is higher (equal) to the cost of separation for the low type sellers times their mass, $T_1(1-x)$.

In the following proposition we present the conditions that have to be satisfied by the parameters to have an equilibrium where high type sellers multihome and low type ones singlehome, with prices $P_1^S = V(q^M)$ and $P_2^S = V(q^H)$. Since for $P_1^S = V(q^M)$ to be an equilibrium it is needed that $(1 - T_2) > V(q^M)$ (see proposition 4) in next proposition we further assume that this condition holds. The result is obtained for the case where $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ satisfies.

Proposition 8 *Let $\beta > \gamma(1 - 2x)$. If $xV(q^H) = V(q^M) > (1 - T_1)(1 - x)$, there is a subgame perfect H multihoming equilibrium with prices $P_1^{S*} = V(q^M)$ and $P_2^{S*} = V(q^H)$ and location of sellers L1HM.*

Proof. See appendix C. ■

Note that condition $xV(q^H) = V(q^M)$ implies that both platforms get the same profits. In this configuration of equilibrium, given the price of the rival, each platform has the possibility of replicating the profit of the other one by setting the other's price (see the proof of the next proposition).

Condition $V(q^M) > (1 - T_1)(1 - x)$ arises as a necessary condition to avoid any deviation of platform 1 to a configuration equilibrium as L1H2.

The proposition that follows characterizes the H multihoming equilibria.

Proposition 9 *In any H multihoming equilibrium both platforms profits are equal.*

Proof. From proposition 4, we know that prices are going to be such that $P_1^S \leq \min\{1 - T_2, V(q^M), P_2^S\}$ and $P_2^S \leq V(q^H)$. Note that in equilibrium platforms will optimally charge prices $P_1^S = \min\{1 - T_2, V(q^M)\}$ and $P_2^S = V(q^H)$. At any other price there exists a profitable deviation for at least one of the platforms. Moreover, both platforms have the possibility of getting the other platform's profits by setting its price. Setting a price $P_2^S = \min\{1 - T_2, V(q^M)\} - \varepsilon$ platform 2 attracts low type sellers and gets the profits that platform 1 obtains in the equilibrium. With a price $P_1^S = V(q^H) - \varepsilon$ platform 1 attracts high type sellers, loses low type sellers and gets the profits of platform 2 in equilibrium. These two deviations will not be profitable whenever profits in equilibrium are equal, which shows the statement. ■

Note that if $1 - T_2 > V(q^M)$, the condition is $V(q^M) = xV(q^H)$ and can be satisfied if and only if $V(q^i)$ is a linear function. On the other hand, if $1 - T_2 < V(q^M)$, the condition that arises is $1 - T_2 = xV(q^H)$ and can be satisfied for any concave function.

Corollary *In any H multihoming equilibrium the price set by the high quality platform is higher than the price set by the medium quality one*

Finally, we study the conditions under which a separating equilibrium can exist. We find that this is the equilibrium less likely to arise and several stringent conditions should be satisfied. The following proposition presents some general conditions that would make a separating equilibrium likely to exist.

Proposition 10 *If x is sufficiently lower than $(1 - T_1)$ a separating equilibrium with location of sellers L1H2 and prices $P_1^{S*} = (1 - T_1)$ and $P_2^{S*} = V(q^H) + M$ may exist.*

Proof. See the appendix C. ■

Now, we characterize any separating equilibrium in the next lemma and proposition.

Lemma 3 In any separating equilibrium, at least one platform makes positive profits, so that a Bertrand like equilibrium never arises.

It follows from condition $T_1 \leq P_2^S - P_1^S < D_q$, which is a necessary condition for separation as shown in proposition 2.

Proposition 11 *In any separating equilibrium:*

- a) *At least one type of sellers makes zero profits;*
- b) *The price of the high quality platform is higher than the price of the low quality platform;*
- c) *Profits of the low quality platform are higher or equal than profits of the high quality platform.¹⁹*

Proof. Let platform 1 be the low quality platform and platform 2 be the high quality one.

a) The corresponding best response prices are: $P_1^S = \min(1 - T_1, P_2^S - T_1)$ and $P_2^S = \min(V(q^H) + M, P_1^S + D_q)$. Now, assume that no type seller is at the participation level, that is $P_1^S = P_2^S - T_1$ and $P_2^S = P_1^S + D_q$. This is only possible if $D_q = T_1$ and we know from proposition 1 that a necessary condition for a separating equilibrium is $D_q > T_1$. So, in equilibrium $P_1^S = 1 - T_1$ or $P_2^S = V(q^H) + M$ as claimed in the proposition.

b) Suppose that platform 1 is the low quality platform and platform 2 the high quality one. If $P_2^S = V(q^H) + M$ the statement b) follows trivially as $V(q^H) + M > 1 - T_1 \geq P_1^S$. If $P_2^S = P_1^S + D_q$, from statement a) we know that $P_1^S = 1 - T_1$. Consequently, $P_2^S = 1 - T_1 + D_q > 1 - T_1 = P_1^S$.

c) The statement follows from two facts. On the one hand, the low quality platform has always the possibility of getting the profits of the high type one (whenever $P_2^S > 1 - T_1$). Setting a price $P_1^S = P_2^S - \varepsilon$, platform 1 attracts sellers type H, loses sellers type L and gets the profits of the high quality platform. It would be a profitable deviation if high quality platform had higher profits than low quality one. On the other hand, given P_1^S , platform 2 can not always replicate the situation of platform 1. This fact explains the asymmetry between profits. ■

Remarks about price competition with multihoming and quality differentiation

¹⁹Damiano and Li (2005) show that in a sequential-move game where platforms compete in prices, under uniform type distribution, the platform that moves first chooses prices such that this platform becomes the low quality one.

Since sellers are allowed to multihome they do not make an "either-or" decision to join a platform. Because of this, competition between platforms is not fierce as in Bertrand games and, in any equilibrium, prices to the sellers are higher than the marginal cost. In fact, prices are close to the monopoly prices. Note that this result is similar to the well known result of Armstrong (2004) ("competitive bottlenecks").²⁰ However, when the reputation effect is absent, even if sellers are allowed to multihome, the unique equilibrium involves a single active platform, where the platform sets a price equal to the marginal cost whenever network effect is weak for high type sellers (i.e., $\beta > \gamma(1 - 2x)$).²¹ The difference between Armstrong's result and ours follows from the timing of the game. Here buyers decide their location after observing sellers' choices. Sellers anticipate that if they group together, they will meet all the buyers. So that, with no reputation effect, the strategy of multihoming will be always a dominated one. When this occurs, platforms have incentives to "undercutting" prices to "steal" sellers. In contrast, whenever the quality effects are present, platforms have two types of pricing strategies.²² First, they can "lowering the own price" to attract sellers. Aside from this usual strategy we identify another one by which the platform with initially lower quality can "increase the own price" to provide a higher level of quality.²³ For instance, given a location LMHM, platform 2 (1) would achieve a location L1HM (L2HM) by setting a price higher than $\min\{T_2, V(q^M)\}$ and lower than $V(q^H)$. By doing so, platform 2 (platform 1) would offer a service of quality q^H , higher than the initial quality q^M , and would attract the highest buyers. Something similar occurs in locations L1HM and L1H2. In both cases, the low quality platform could increase its price to expel the low type sellers while keeping the high types. So that, this platform becomes of the same quality than its rival. Once this is the case location LOH1 might emerge.

4. Charges are zero to the sellers

Consider a situation where platforms do not set positive charges to the sellers but do to the buyers. A good example are software platforms that set access charge to user and receive scarce revenues from developers that use their intellectual property²⁴. Other examples are real state and travel agencies that set charges only to the buyers. Some web pages that search for hotels and flights charge prices only to the buyers. In these last examples, prices are charged if

²⁰It says that the singlehoming side is treated favorably compared with the multihoming side. For computational simplicity, we have assumed that prices are zero for the singlehoming side, but we conjecture that it may arise as an equilibrium result.

²¹Recall, from proposition 6 that prices are $P_1^S = \min\{T_1, [F(\gamma(1 - 2x)) - F(\beta^H)]\}$, $P_2^S = 0$ if $\beta < \gamma(1 - 2x)$.

²²Although this kind of analyses is not totally correct in a static environment with simultaneous competition, it is useful to give an intuition about the driving forces.

²³Note that this strategy is similar to the "overtaking strategy" in Damiano and Li (2005), although the our is not precisely an overtaking one.

²⁴In a survey about software platforms, Evans, et.al. (2004) remark: "Microsoft has earned virtually all of its revenue from end-users...and thus earned only minor revenue from licensing software tools".

a transaction takes place so that they do not completely fit with our model of prices by access but they are a good illustration of the intuition.

We maintain the assumption that buyers singlehome and sellers are allowed to multihome. This is a reasonable assumption for the case of the software platforms, given that, in general, end-users use a single software platform in their computers while developers can write for several platforms.

In this setting, platforms set prices to the buyers in the first stage, sellers observe prices set to the buyers and infer buyers' locations in the second stage, finally, buyers observe prices, sellers' locations, infer quality and choose the platform they visit.

4.1 The buyers' problem

Two active platforms with $q^i > q^j$. For each (P_i^B, P_j^B) and each (N_i^S, N_j^S) we define θ_1^* as the buyer who is indifferent between visiting platform i and platform j , i.e.,

$$\theta_1^* = \min\{\bar{\theta}, \gamma \frac{(N_j^S - N_i^S)}{(q^i - q^j)} + \frac{(P_i^B - P_j^B)}{(q^i - q^j)}\} \quad (6)$$

and θ_0^* as the indifferent buyer between visiting platform j and no visiting any platform

$$\theta_0^* = \max\{\underline{\theta}, \frac{(P_j^B - \gamma N_j^S)}{q^j}\} \quad (7)$$

Then, the number of buyers in platform i is $N_i^B = 1 - F(\theta_1^*)$ and the corresponding in platform j is $N_j^B = F(\theta_1^*) - F(\theta_0^*)$.

Two active platforms with $q^i = q^j$. Two configurations may arise. If $\gamma N_i^S - P_i^B > \max\{\gamma N_j^S - P_j^B, 0\}$, then $N_i^B = 1$ and $N_j^B = 0$, whereas if $\gamma N_i^S - P_i^B = \gamma N_j^S - P_j^B > 0$, then $N_i^B = N_j^B = \frac{1}{2}$.

A single active platform j with q^j . There is a θ_0^* that represents the indifferent buyer between visiting platform j and no visiting it and the mass of buyers that go to the platform j is $N_j^B = 1 - F(\theta_0^*)$.

4.2 The sellers' problem

As in the previous section, at the second stage we consider the Strong Nash refinement.

Proposition 12 *Let $[P_1^B, P_2^B]$ be the vector of prices set by the platforms to the buyers. There is a unique Strong Nash Equilibrium in the sellers subgame, the global multihoming equilibrium (location LMHM).*

Proof. For both type of sellers the strategies of going to only one platform, any of them, is dominated by the strategy of making multihoming. ■

4.3 The platforms' problem

Proposition 13 *There is a unique subgame perfect Nash Equilibrium, the global multihoming equilibrium (location LMHM) with prices $P_1^B = P_2^B = 0$ and the number of buyers $N_1^B = N_2^B = \frac{1}{2}$.*

Proof. Given that sellers multihome both platforms have the same quality q^M . The platform that sets the lower price gets the total market of buyers so that Bertrand competition shows the result. ■

Note that once we have imposed prices zero to the side allowed to multihome, competition for the side that singlehome is very strong and prices equal marginal costs.

5. Welfare

Consider the total welfare as the sum of the buyers' surplus, the sellers' surplus and the platforms' profits. Denote with Λ the aggregate of the sellers' surplus and the platforms' profits and note that, for any configuration where $P_1^B = P_2^B = 0$ this is equal to,

$$\Lambda = \left[V(q^i) + N_i^B(\beta^H) \right] N_i^{SH} + N_i^B N_i^{SL} + \left[V(q^j) + N_j^B(\beta^H) \right] N_j^{SH} + N_j^B N_j^{SL}.$$

We present the particular values that Λ takes in the configurations that we have worked in.

In a dominant firm configuration this is equal to

$$\Lambda^{LiHi} = [V(q^M) + M]x + (1-x), \quad i \neq M.$$

In the case of a separating equilibrium the value of Λ is

$$\begin{aligned} \Lambda^{LiHj} &= [V(q^H) + M]x + (1-x)(1-T_1), \quad i \neq j \neq M, \text{ if } \beta^H > \gamma(1-2x) \text{ and} \\ \Lambda^{LiHj} &= [V(q^H) + T_1]x + (1-x)(1-T_1), \quad i \neq j \neq M, \text{ if } \beta^H < \gamma(1-2x). \end{aligned}$$

The particular value in a H multihoming equilibrium is

$$\Lambda^{LiHM} = [V(q^H) + V(q^M) + M]x + (1-x)(1-T_2), \quad i \neq M.$$

Finally, the value of Λ in a global multihoming equilibrium is

$$\Lambda^{LMHM} = [2V(q^M) + M]x + (1-x).$$

Note that, for any β^H ,

$$\Lambda^{LiHM}(i \neq M) > \Lambda^{LiHj}(i \neq j \neq M),$$

this is that the aggregate profits of the sellers and platforms in a H multihoming equilibrium is always higher than the corresponding to a separating equilibrium.

It also occurs that the aggregate profits of the sellers' and platforms in an equilibrium configuration as LMHM are, in any case, higher than in a dominant firm equilibrium, this is,

$$\Lambda^{LMHM} > \Lambda^{LiHi} (i \neq M).$$

Finally, note that whenever $\frac{D_q}{T_2} > \frac{1-x}{x}$, condition that is implied by $\frac{D_q}{T_1} \geq \frac{1-x}{x}$,

$$\Lambda^{LiHM} (i \neq M) > \Lambda^{LMHM}.$$

Now, consider the buyers' surplus. Note that they obtain the same surplus under an equilibrium configuration L1H1 (or L2H2) than under an equilibrium configuration LMHM and this is

$$\int_{\underline{\theta}}^{\bar{\theta}} [uq^M + \gamma] f(u) du$$

where $E(\theta)$ is the expected value of θ .

Given that $\Lambda^{LMHM} > \Lambda^{LiHi} (i \neq M)$, we conclude that welfare in a LMHM configuration is always higher than in a $LiHi (i \neq M)$ configuration (note that this is only explained by the fact that high type sellers get a positive profit $2V(q^M)$ in the case of LMHM and only $V(q^M)$ in the case of L1H1).

The buyers' surplus of a configuration as L1H2 (or L2H1) is,

$$\int_{\underline{\theta}}^{\gamma(1-2x)} [uq^L + \gamma(1-x)] f(u) du + \int_{\gamma(1-2x)}^{\bar{\theta}} [uq^H + \gamma x] f(u) du,$$

and the buyers' surplus of a configuration as L1HM (or L2HM) is,

$$\int_{\underline{\theta}}^{\gamma} [uq^M + \gamma] f(u) du + \int_{\gamma}^{\bar{\theta}} [uq^H + \gamma x] f(u) du.$$

If we compare the buyers' surplus in both configurations, we note that buyers of the type in the interval $[\underline{\theta}, \gamma]$ are strictly better off under a configuration $LiHM (i \neq M)$ than under one like $LiHj (i \neq j \neq M)$. In addition, buyers of the type in the interval $[\gamma, \bar{\theta}]$ are indifferent between both configurations.

Taking into account that $\Lambda^{LiHM} (i \neq M) > \Lambda^{LiHj} (i \neq j \neq M)$, it follows that a H multihoming configuration yields a higher total level of welfare than a separating one.

From the comparison of the buyers' surplus in a global multihoming configuration and the corresponding in a H multihoming one, we know that buyers of the type in the interval $[\underline{\theta}, \gamma]$ are indifferent while those of the type in the interval $[\gamma, \bar{\theta}]$ will prefer the configuration $LiHM (i \neq M)$.

We can conclude that under the condition $\frac{D_q}{T_1} \geq \frac{1-x}{x}$, among the four configurations that we have analysed the one that generates the higher total level of welfare is the H multihoming one.

Remember that prices to sellers equal to the marginal cost, *i.e.*, $P_1^S = P_2^S = 0$, would lead to a global multihoming equilibrium with $P_1^B = P_2^B = 0$. It shows that if the total price $(P_i^S + P_i^B)$ set by the platforms equals the marginal cost, the location that yields the highest welfare is not attained.

6. Conclusions

We have considered a model of competition between two-sided platforms where each side of the market not only care about the size of the other side, but also about the type of its members. When buyers and sellers interact through the platforms there are network and quality effects operating from one market to the other. There is also an own side effect on the sellers' side.

Although the network effects and the ex-ante symmetric platforms, we find equilibria with more than a single active platform. Moreover, in some of them the resulting platforms are asymmetric in the prices they set, the type of customers they house and the quality of the service they provide.

That sellers care about the type of other sellers in the platform is a crucial assumption to have equilibria configurations with more than one platform. The heterogeneity about quality on the buyers' side plays also an important role for the results, in particular, to the existence of equilibria with asymmetric platforms.

Because we have allowed the sellers to multihome, we find that they are charged above the marginal cost, if not with monopoly prices.

Finally, the equilibrium where sellers separate by type is the most demanding in terms of parameter values. The equilibrium where low type sellers singlehome and high type multihome, so that one platform provides a high quality service while the other one provides a medium level of quality, is the equilibrium that yields the highest welfare, among the equilibria analyzed. Nevertheless, this equilibrium may not always exist.

Appendix A

Proof of proposition 2

Proof. We show that prices satisfying the conditions stated in proposition 2 imply an equilibrium configuration L1H2 by iterated elimination of dominated strategies. Under prices $P_1^S \leq 1 - T_1$ and $P_2^S \leq V(q^H) + M$, strategies L0 and H0 are eliminated. Given that $P_2^S - P_1^S \geq T_1$, L1 dominates L2 and LM. Finally, the best response to L1 by high type sellers is H2. ■

Proof of proposition 3

Proof. First, we prove that in any LMHM configuration of equilibrium, prices satisfy $\min\{T_2, V(q^M)\} \geq \max\{P_1^S, P_2^S\}$ and $P_1^S + P_2^S \leq 1$. The profits of the sellers are $U_1^H + U_2^H = 2V(q^M) + M - P_1^S - P_2^S$ and $U_1^L + U_2^L = 1 - P_1^S - P_2^S$, where $P_1^S + P_2^S \leq 1$ and $P_1^S + P_2^S \leq 2V(q^M) + M$ must hold to ensure sellers' participation. High type sellers will not deviate whenever

$$2V(q^M) + M - P_1^S - P_2^S \geq \max\{V(q^M) + M - P_1^S, V(q^M) + M - P_2^S\}$$

which requires $P_1^S \leq V(q^M)$ and $P_2^S \leq V(q^M)$ to hold. Similarly, low type sellers will not deviate whenever

$$1 - P_1^S - P_2^S \geq \max\{(1 - T_2) - P_1^S, (1 - T_2) - P_2^S\}$$

which requires $P_1^S \leq T_2$ and $P_2^S \leq T_2$. Thus, the first implication follows.

Now, by iterated elimination of dominated strategies we show that prices such that $\min\{T_2, V(q^M)\} \geq \max\{P_1^S, P_2^S\}$ and $P_1^S + P_2^S \leq 1$ ensure the existence of a global multihoming equilibrium as claimed. Trivially, strategies L0 and H0 are eliminated given that participation conditions are guaranteed. Given that $\max\{P_1^S, P_2^S\} \leq V(q^M)$, HM dominates H1 and H2. Finally, the best reply of sellers type L to HM strategy is LM under prices $\max\{P_1^S, P_2^S\} \leq T_2$. ■

Proof of proposition 4

Proof. Initially, we prove that in any L1HM prices satisfy $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$.

First, consider the case $\beta^H > \gamma(1 - 2x)$. In a H multihoming equilibrium configuration as L1HM, sellers' profits are given by $U_M^H = V(q^H) + V(q^M) + M - P_1^S - P_2^S$ and $U_1^L = (1 - T_2) - P_1^S$. Participation constraints require $P_1^S \leq 1 - T_2$ and $P_1^S + P_2^S \leq V(q^H) + V(q^M) + M$ to hold. High type sellers will not deviate whenever $P_1^S \leq V(q^M)$ and $P_2^S \leq V(q^H)$ hold. Similarly, low type sellers will not deviate whenever $P_2^S - P_1^S \geq 0$ and $P_2^S \geq T_2$ are satisfied.

Now, consider the case $\beta^H < \gamma(1 - 2x)$. The conditions for the low type sellers do not change and high type sellers will not deviate whenever they do not want to deviate to any of the singlehoming strategies, which requires $P_1^S \leq V(q^M) + M - T_1$ and $P_2^S \leq V(q^H)$ to hold. Given that $M > T_1$ when $\beta^H < \gamma(1 - 2x)$, condition $P_1^S \leq V(q^M) + M - T_1$ is implied by $P_1^S \leq V(q^M)$.

Now, we prove that under prices that satisfy $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$, only a L1HM equilibrium can arise. As in previous proofs we obtain the result by iterated elimination of dominated strategies. The strategies L0 and H0 are trivially eliminated. Given that $P_1^S \leq V(q^M)$, HM dominates H2. And then, L1 dominates L2 and LM. Finally, the best response of high type sellers to L1 is HM.

Thus, the conditions $P_2^S - P_1^S \geq 0$, $P_1^S \leq \min\{1 - T_2, V(q^M)\}$ and $T_2 \leq P_2^S \leq V(q^H)$ are necessary and sufficient to ensure the existence of an equilibrium configuration L1HM, as claimed. ■

Proof of proposition 5

Proof. First, we prove that in any L1H1 equilibrium prices satisfy $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + M\}$ and $P_2^S \geq V(q^H)$.

Assume initially that $\beta^H > \gamma(1 - 2x)$. In a dominant firm equilibrium configuration as L1H1, sellers' profits are given by $U_1^H = V(q^M) + M - P_1^S$ and $U_1^L = 1 - P_1^S$, where $P_1^S \leq \min\{1, V(q^M) + M\}$ ensure that profits above are positive. High type sellers will not deviate to platform 2 whenever $P_2^S - P_1^S \geq D_q$

and will not deviate to a multihome strategy whenever $P_2^S \geq V(q^H)$ is satisfied. Similarly, low type sellers will not deviate whenever $P_2^S - P_1^S \geq -T_1$. Note that this condition is implied by $P_2^S - P_1^S \geq D_q$.

Assume now $\beta^H < \gamma(1 - 2x)$. The conditions for the low type sellers do not change and high type sellers' profits are given by $U_1^H = V(q^M) + T_1 - P_1^S$. They will not deviate to platform 2 whenever $P_2^S - P_1^S \geq D_q - [F(\gamma(1 - 2x)) - F(\beta^H)]$ holds. This condition is implied by $P_2^S - P_1^S \geq D_q$. The condition of no deviation to a multihome strategy is the same than in the case $\beta^H > \gamma(1 - 2x)$.

Now, we prove that under prices that satisfy $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + M\}$ and $P_2^S \geq V(q^H)$ only a L1H1 equilibrium will arise. The implication is proved by iterated elimination of dominated strategies. As in previous proofs, strategies L0 and H0 are trivially eliminated. Under prices $P_2^S - P_1^S \geq D_q$, strategy H1 dominates H2 and under condition $P_2^S \geq V(q^H)$, HM is dominated by H1. Finally, given strategy H1, the best reply of low type sellers is L1.

Consequently, the necessary and sufficient conditions for a dominant platform equilibrium configuration are $P_2^S - P_1^S \geq D_q$, $P_1^S \leq \min\{1, V(q^M) + M\}$ and $P_2^S \geq V(q^H)$ as claimed. ■

Appendix B

NE in mixed strategies of the sellers' subgame

B1) We present the set of Nash Equilibria in mixed strategy of the sellers' subgame when $P_2^S = V(q^H)$ and the price of platform 1 belongs to the interval $1 - T_1 < P_1^S < 1$.

Under these prices (since $V(q^H) > q^H = 1$), iterative elimination of dominated strategies²⁵ shows that low type sellers can only randomize between strategies L1 and L0 and high type sellers can only randomize between strategies H1 and H2.

Denoting by a to the probability of playing L1 and by y to the probability of playing H1, high sellers' expected utility is given by

$$U^H(a; y) = y(V(q^H) - aD_q - P_1^S).$$

The best reply by H involves

$$BR^H(a; (y, b)) = \begin{cases} y = 1 & \text{if } V(q^H) - aD_q - P_1^S > 0 \\ y = 0 & \text{if } V(q^H) - aD_q - P_1^S < 0 \\ c \in [0, 1] & \text{if } V(q^H) - aD_q - P_1^S = 0. \end{cases}$$

And the low type sellers' utility is given by

$$U^L(a; y) = a(1 - P_1^S - T_1 + T_1y),$$

²⁵We find that L1 dominates LM, then H1 weakly dominates HM, L0 dominates L2 and finally, H2 dominates H0.

and the corresponding best reply by low type sellers is

$$BR^L(a; (y, b)) = \begin{cases} a = 1 & \text{if } (1 - P_1^S - T_1 + T_1 y) > 0 \\ a = 0 & \text{if } (1 - P_1^S - T_1 + T_1 y) < 0 \\ a \in [0, 1] & \text{if } (1 - P_1^S - T_1 + T_1 y) = 0. \end{cases}$$

Note that, given $P_2^S = V(q^H)$, whenever P_1^S belongs to the interval $[1, V(q^H)]$, the equilibrium configuration is L0H1. Similarly, if P_1^S belongs to the interval $[V(q^M), 1 - T_1]$, the equilibrium configuration is L1H2. Consequently, we only need to derive the set of NE in mixed strategies for the interval of prices $1 - T_1 < P_1^S < 1$.

Lemma 4: If $1 - T_1 < P_1^S < 1$, high type sellers randomize between strategies H1 and H2 with probabilities $y = \frac{1}{T_1} (P_1^S - (1 - T_1))$ and $1 - y$ respectively, whereas low type sellers randomize between strategies L1 and L0 with probabilities $a = \frac{V(q^H) - P_1^S}{D_q}$ and $1 - a$. The size of the market of platform 1 is given

by

$$\left(\frac{V(q^H) - P_1^S}{D_q} \right) (1 - x) + \left(\frac{1}{T_1} (P_1^S - (1 - T_1)) \right) x \quad (8)$$

Proof. The value $a = \frac{V(q^H) - P_1^S}{D_q}$ leaves the high type sellers indifferent between strategies H1 and H2. Analogously, the value $y = \frac{1}{T_1} (P_1^S - (1 - T_1))$ leaves the low type sellers indifferent between strategies L1 and L0. ■

Note that, whenever $\frac{D_q}{T_1} \geq \frac{1-x}{x}$, markets defined by equation (9), (10), (11) and (8) are non-decreasing in prices.

B2) We present the set of Nash Equilibria in mixed strategy of the sellers' subgame when $P_1^S = 1 - T_1$ and the price of platform 2 belongs to the interval $V^M \leq P_2^S \leq 1$.

The set of equilibria is computed assuming that LM is a dominated strategy (it occurs if $1 - T_1 > T_1$ or if $V(q^M) > T_1$).

In addition, this set of equilibria exists when parameters satisfy also the following conditions: $2(1 - T_1) < V(q^H) + V(q^M) < 2 - T_1$.

We denote by a the probability of the low type sellers playing strategy L1 and consequently $(1 - a)$ the probability of L2. We denote by y , b and $c = 1 - y - b$ the probabilities of the high type sellers playing strategies H1, H2 and HM, respectively.

Given $P_1^S = 1 - T_1$, the high type sellers' expected utility is given by

$$\begin{aligned} U^H(a; (y, b)) &= M + y(T_1 + V(q^H)(1 - a) + aV(q^M) - 1) + \\ &\quad b(V(q^M)(1 - a) + aV(q^H) - p_2) + c(V(q^M) + V(q^H) + T_1 - 1 - p_2) \\ &= M + yz_1 + bz_2 + cz_3 \end{aligned}$$

The best reply by H involves

$$BR^H(a; (y, b)) = \begin{cases} y = 1 & \text{if } z_1 > \max(0, z_2, z_3) \\ b = 1 & \text{if } z_2 > \max(0, z_1, z_3) \\ c = 1 & \text{if } z_3 > \max(0, z_1, z_2). \end{cases}$$

Given $P_1^S = 1 - T_1$, the high type sellers' expected utility is given by

$$U^L(a; (y, b)) = a(yT_1 + T_1 - bT_1 - 1 + p_2) - yT_1 + 1 - T_2 + T_2y + T_2b - p_2$$

and the corresponding best reply by low type sellers is

$$BR^L(a; (y, b)) = \begin{cases} a = 1 & \text{if } (1 + y - b)T_1 - 1 + p_2 > 0 \\ a = 0 & \text{if } (1 + y - b)T_1 - 1 + p_2 < 0 \\ a \in [0, 1] & \text{if } (1 + y - b)T_1 - 1 + p_2 = 0. \end{cases}$$

Using the best reply functions above, we next show the set of Nash equilibria in mixed strategies.

Lemma 5: Along the interval $V(q^H) + V(q^M) - (1 - T_1) < P_2^S \leq 1$ there exists a set of mixed strategy Nash Equilibria where low type sellers randomize between strategies L1 and L2 and high type sellers randomize between strategies H1 and H2 with probabilities $a = \left(\frac{1}{2} \frac{D_q + P_2^S - (1 - T_1)}{D_q}\right)$, $y = \left(\frac{1}{2} \frac{1 - P_2^S}{T_1}\right)$ and $b = 1 - y$. The platform 2's market in this interval is equal to

$$\left(\frac{1}{2} \frac{D_q - P_2^S + (1 - T_1)}{D_q}\right) (1 - x) + \left(\frac{1}{2} \frac{T_1 - (1 - T_1) + P_2^S}{T_1}\right) x \quad (9)$$

Proof. If $V(q^H) + V(q^M) - (1 - T_1) < P_2^S$, then $z_3 < 0$ so that strategy HM is dominated. At $a = \left(\frac{1}{2} \frac{D_q + P_2^S - (1 - T_1)}{D_q}\right)$ sellers type H are indifferent between H1 and H2. Similarly, $y = \left(\frac{1}{2} \frac{1 - P_2^S}{T_1}\right)$ is the probability that makes low type sellers indifferent between L1 and L2. The size of the market follows trivially

from the probabilities above. ■

Lemma 6: Along the interval $(1 - T_1) < P_2^S < V(q^H) + V(q^M) - (1 - T_1)$, sellers type L randomize between strategies L1 and L2 whereas sellers type H randomize between strategies H2 and HM with probabilities $a = \frac{(V(q^H) - (1 - T_1))}{D_q}$, $b = \left(\frac{1}{T_1} (P_2^S - (1 - T_1))\right)$ and $c = 1 - b$.²⁶ The platform 2's market in this interval is equal to

$$\left(\frac{(1 - T_1) - V(q^M)}{D_q}\right) (1 - x) + x \quad (10)$$

²⁶If the price is $P_2^S = V(q^H) + V(q^M) - (1 - T_1)$, sellers type L randomize between L1 and L2 whereas sellers type H randomize with positive weights in their three strategies.

Proof. At $a = \frac{V(q^H) - (1 - T_1)}{D_q}$, sellers type H are indifferent between their three possible strategies as $z_1 = z_2 = z_3$. The values that leave sellers type L indifferent between L1 and L2 are $b = \left(\frac{1}{T_1} (P_2^S - (1 - T_1))\right)$ and $c = 1 - b$, which trivially follows from $BR^L(a; (y, b))$. ■

Lemma 7: If $V^M < P_2^S < (1 - T_1)$ low type sellers randomize between strategies L1 and L2 with $a = \frac{(P_2^S - V(q^M))}{D_q}$, and sellers type H randomize between strategies H1 and HM with probabilities $y = \frac{(1 - T_1 - P_2^S)}{T_1}$ and $c = 1 - y$.²⁷ The size of the market of platform 2 in this interval of prices is

$$\left(\frac{V(q^H) - P_2^S}{D_q}\right)(1 - x) + \left(\frac{T_1 - (1 - T_1) + P_2^S}{T_1}\right)x \quad (11)$$

Proof. Whenever the value of a is smaller than $\frac{V(q^H) - (1 - T_1)}{D_q}$, the best response of the high type sellers is $b = 0$. The value $y = \frac{(1 - T_1 - P_2^S)}{T_1}$ leaves low sellers indifferent between strategies L1 and L2. The platform 2's market trivially follows. ■

Appendix C

Proof of proposition 7

Proof. We first show that there is no profitable deviation by platform 2 given $P_1^{S*} = \min\{T_2, V(q^M)\}$.

At the candidate equilibrium platform 2's profits are $\Pi_2^* = \min\{T_2, V(q^M)\}$ provided that $\min\{T_2, V(q^M)\} \leq \frac{1}{2}$, which ensures low type sellers participation.

As shown in proposition 3 no other price by platform 2 will yield a higher profit among the prices that induce global multihoming.

Consequently, we only need to check for deviations to a higher price that would induce sellers' location L1HM provided that $P_2^S < V(q^H)$. Such a deviation is not profitable if and only if

$$P_2^S x < \min\{T_2, V(q^M)\}.$$

As $P_2^S < V(q^H)$, a sufficient condition to deter this deviation is given by $V(q^H)x < \min\{T_2, V(q^M)\}$. If $T_2 > V(q^M)$, condition $V(q^H)x < V(q^M)$ is guaranteed by the concavity of $V(q^i)$. If $T_2 < V(q^M)$, the necessary condition is $V(q^H)x < T_2$.

Similar arguments apply to platform 1 deviations which show our claim. ■

²⁷If $P_2^S = (1 - T_1)$ there is a set of NE in mixed strategies where sellers type L randomize between L1 and L2 with probabilities $a \in \left(\frac{(1 - T_1) - V(q^M)}{D_q}, \frac{V(q^H) - (1 - T_1)}{D_q}\right)$ whereas high type sellers play the strategy HM.

If $P_2^S = V(q^M)$ sellers type L play strategy L2 and high type sellers randomize between strategies H1 and HM with probabilities $y \in [0, \frac{(1 - T_2 - P_2^S)}{T_1})$.

Proof of proposition 9

Proof. At the candidate equilibrium platforms' profits are $\Pi_1^* = V(q^M)$ and $\Pi_2^* = V(q^H)x$.

We start analyzing deviations by platform 2 given $P_1^{S*} = V(q^M)$. The set of possible deviations can be divided into three groups:

1) Deviations to prices that guarantee a location of sellers L1HM. These deviations are not profitable as P_2^{S*} is the best price that platform 2 can set among those that yield locations L1HM.

2) Deviations to a lower price $P_2^S \leq V(q^M)$ to attract the low type sellers in order to get the location L2HM.

The deviation is not profitable whenever

$$V(q^M) \leq V(q^H)x. \quad (12)$$

Note that this condition contradicts concavity of function $V(q^i)$, and only can be satisfied if and only if $V(q^i)$ is linear, so that $V(q^M) = V(q^H)x$.

3) Deviations to a higher price $P_2^S > V(q^H)$. These are never profitable deviations given that these prices would lead to a location of sellers L1H1 that implies $\Pi_2^* = 0$.

Now, we analyze deviations by platform 1 given $P_2^{S*} = V(q^H)$. There are three groups of potential deviations:

1) Deviations to prices that guarantee a location of sellers L1HM. These deviations are not profitable because, P_1^{S*} is the maximum price that platform 1 can charge.

2) Deviations to a lower price $P_1^S < V(q^M)$. Given that platform 1 has the total number of sellers, to set a lower price will never be a profitable deviation.

3) Deviations to a higher price. Three relevant intervals arise:

Interval 1: $V(q^M) < P_1^S \leq 1 - T_1$.

Any price of this interval would lead to location of sellers L1H2, so that platform 1 will not deviate if and only if

$$V(q^M) > (1 - T_1)(1 - x). \quad (13)$$

Interval 2: $1 - T_1 < P_1^S < 1$

At prices of this interval sellers play mixed strategies in equilibrium (see B2 in appendix B). In particular, platform 1 gets the low type sellers with probability a and gets the high type sellers with probability y . Platform 1's market is given by (8) that under the condition $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ is non-decreasing in P_1^S . Consequently profits at these prices are bounded above by the profits at $P_1^S = 1$. No deviation will take place if

$$V(q^M) > \left(\frac{V(q^H) - 1}{D_q} \right) (1 - x) + x \quad (14)$$

Interval 3: A price in the interval $1 \leq P_1^S \leq V(q^H)$ would lead to a location L0H1 (see B1 in appendix B) and platform 1 would not deviate if

$$V(q^M) \geq V(q^H)x. \quad (15)$$

Note that (15) implies (14), as $V(q^H) > 1$. Furthermore, (12) and (15) can only hold if $V(q^M) = V(q^H)x$.

Consequently, if (12), (13) and (15) hold then, there is a Subgame Perfect Equilibrium configuration with prices $P_1^{S*} = V(q^M)$ and $P_2^{S*} = V(q^H)$ and location L1HM as we claimed. ■

Proof of proposition 11

Proof. At the candidate equilibrium platform's profits are $\Pi_1^* = (1 - T_1)(1 - x)$ and $\Pi_2^* = (V(q^H) + M)x$.

We first analyze deviations by platform 2 given $P_1^{S*} = 1 - T_1$. These deviations can be divided into three groups:

1) Deviations to prices that guarantee sellers' separation, i.e., to prices in the interval $[1, D_q + 1 - T_1]$. These deviations are not profitable as P_2^{S*} is the monopoly price.

2) Platform 2 can deviate to a lower price $P_2^S \leq 1$ to attract more sellers and obtain higher benefits. There are 3 intervals of prices to be considered. In all of them sellers play mixed strategies in equilibrium (see B2 in Appendix B):

Interval 1: $V(q^H) + V(q^M) - (1 - T_1) < P_2^S \leq 1$

As shown in lemma 4 of appendix B, at these prices low type sellers go to platform 2 with probability $(1 - a)$ and high type sellers with probability b , hence the market of platform 2 in this interval is (9).

Interval 2: $(1 - T_1) < P_2^S \leq V(q^H) + V(q^M) - (1 - T_1)$

At these prices lemma 5 shows that platform 2 gets all the high type sellers and gets the low type with probability $(1 - a)$, so that the market of platform 2 is (10). Note that (10) is higher than (9).

A sufficient condition for no deviation to any price in the two previous intervals by platform 2 is to evaluate profits at $P_2^S = 1$. The condition that arises is

$$(V(q^H) + M)x \geq \left(\frac{(1 - T_1) - V(q^M)}{D_q} \right) (1 - x) + x \quad (16)$$

Interval 3: $V(q^M) < P_2^S \leq (1 - T_1)$

At these prices from lemma 6 we know that platform 2 gets the low type sellers with probability $(1 - a)$ and the high type sellers with probability $(1 - y)$. The relevant market of platform 2 for this interval is given by (11). If the market in this interval is non-decreasing in P_2^S , i.e., condition $\frac{D_q}{T_1} \geq \frac{1-x}{x}$ holds, a sufficient condition for no deviation arises. If $\Pi_2^* \geq [\text{Market}(P_2^S = 1 - T_1)](1 - T_1)$, i.e.,

$$(V(q^H) + M)x \geq \left[\left(\frac{V(q^H) - (1 - T_1)}{D_q} \right) (1 - x) + x \right] (1 - T_1) \quad (17)$$

If $P_2^S \leq V(q^M)$ the condition that avoids any deviation to a location L2HM is

$$(V(q^H) + M)x \geq V(q^M) \quad (18)$$

3) Deviations to a higher price $P_2^S \geq D_q + 1 - T_1$. These deviations are not profitable given that they would lead to a location of sellers L1H1 that implies $\Pi_2 = 0$.

Now, consider deviations by platform 1 given $P_2^{S*} = V(q^H) + M$. These deviations can be divided into three groups:

1) Deviations to prices that guarantee sellers' separation, i.e., to prices in the interval $[V(q^H) + M - D_q, V(q^H) + M - T_1]$. These deviations are not profitable as P_2^{S*} is the monopoly price.

2) Platform 1 can deviate to a lower price $P_1^S \leq V(q^H) + M - D_q = V(q^M) + M$. These prices would lead to a location of sellers L1H1 that implies $\Pi_1 = V(q^M) + M$. This deviation is not profitable whenever

$$V(q^M) + M < (1 - T_1)(1 - x) \quad (19)$$

3) Deviations to a higher price $P_1^S \geq V(q^H) + M - T_1$. The best deviation can do is attracting type H sellers while type L are lost (location L0H1). The maximum price that gets it is $P_1^S = V(q^H) + M - \varepsilon$ and platform 1 will not deviate if and only if

$$(V(q^H) + M)x \leq (1 - T_1)(1 - x) \quad (20)$$

Note that (17) and (20) are compatible if and only if $\frac{(1-T_1)-V(q^M)}{D_q} > \frac{x}{1-x}$

Note that (16) and (20) are compatible if and only if $\frac{(1-T_1)(D_q-1)+V(q^M)}{D_q} > \frac{x}{1-x}$.

Condition (20) is compatible with $V(q^H) > 1$ if and only if $\frac{(1-T_1)}{(1+M)} > \frac{x}{1-x}$.

The common feature of the conditions are that x must be sufficiently low and $(1 - T_1)$ sufficiently high to hold. ■

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